

## 3.7.2 Mocniny II

**Předpoklady:** 030701

**Př. 1:** Dopln tabulku.

$x$	-3	-0,2		-30				0,5	
$x^3$			0,001		8000	0,027	-64		8000000

$x$	-3	-0,2	0,1	-30	20	0,3	-4	0,5	200
$x^3$	-27	-0,008	0,001	-27000	8000	0,027	-64	0,125	8000000

**Př. 2:** Odstraň zlomky a případně zjednoduš.

a)  $\frac{1}{2^3}$       b)  $\frac{1}{a^2}$       c)  $\frac{1}{3^2 \cdot 3^4}$       d)  $\frac{a^3}{a^5}$       e)  $\frac{a^2}{a^7 \cdot a}$

a)  $\frac{1}{2^3} = 2^{-3}$       b)  $\frac{1}{a^2} = a^{-2}$       c)  $\frac{1}{3^2 \cdot 3^4} = \frac{1}{3^6} = 3^{-6}$

d)  $\frac{a^3}{a^5} = \frac{1}{a^2} = a^{-2}$  (rychleji  $\frac{a^3}{a^5} = a^{3-5} = a^{-2}$ )

e)  $\frac{a^2}{a^7 \cdot a} = \frac{a^2}{a^8} = \frac{1}{a^6} = a^{-6}$  (rychleji  $\frac{a^2}{a^7 \cdot a} = a^{2-7-1} = a^{-6}$ )

**Př. 3:** Zapiš jako jednu mocninu.

a)  $4^7 \cdot 4^2$       b)  $2 \cdot 2^7 \cdot 2^{-2}$       c)  $\frac{3^5}{3^3}$       d)  $\frac{4^4}{4^{-3}}$

e)  $2^k \cdot 2^2$       f)  $5 \cdot 5^k \cdot 5^{k+1}$       g)  $\frac{x^{k+2}}{x^3}$       h)  $\frac{2^{2k+3}}{2^{1-k}}$

a)  $4^7 \cdot 4^2 = 4^{7+2} = 4^9$       b)  $2 \cdot 2^7 \cdot 2^{-2} = 2^{1+7-2} = 2^6$       c)  $\frac{3^5}{3^3} = 3^{5-3} = 3^2$

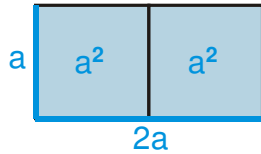
d)  $\frac{4^4}{4^{-3}} = 4^{4-(-3)} = 4^7$       e)  $2^k \cdot 2^2 = 2^{k+2}$       f)  $5 \cdot 5^k \cdot 5^{k+1} = 5^{1+k+k+1} = 5^{2k+2}$

g)  $\frac{x^{k+2}}{x^3} = x^{k+2-3} = x^{k-1}$       h)  $\frac{2^{2k+3}}{2^{1-k}} = 2^{2k+3-(1-k)} = 2^{2k+3-1+k} = 2^{3k+2}$

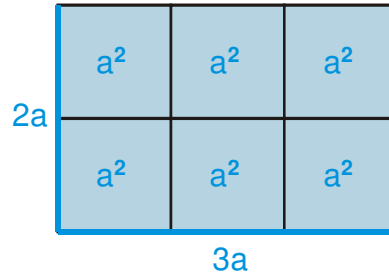
**Př. 4:** Vypočti a graficky znázorni.

- a)  $a \cdot 2a$       b)  $2a \cdot 3a$       c)  $(3a)^2$       d)  $a \cdot 2b$       e)  $a \cdot 2a \cdot 2a$

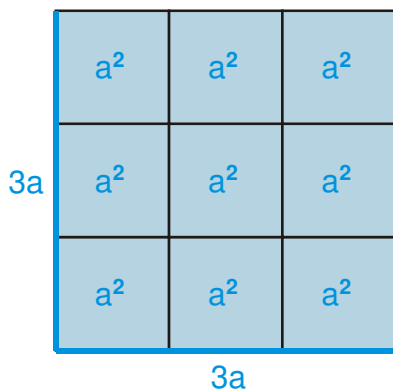
a)  $a \cdot 2a = 2 \cdot a \cdot a = 2a^2$



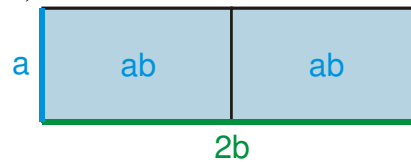
b)  $2a \cdot 3a = 2 \cdot 3 \cdot a \cdot a = 6a^2$



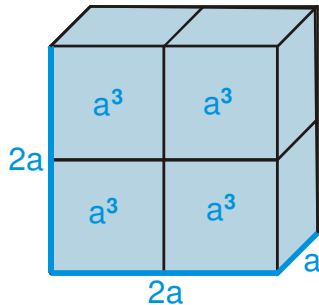
c)  $(3a)^2 = 3a \cdot 3a = 3 \cdot 3 \cdot a \cdot a = 9a^2$



d)  $a \cdot 2b = 2 \cdot a \cdot b = 2ab$



e)  $a \cdot 2a \cdot 2a = 2 \cdot 2 \cdot a \cdot a \cdot a = 4a^3$



**Př. 5:** Vypočti bez kalkulačky.

- a)  $2^4 \cdot 5^4$       b)  $4^2 \cdot 5^2$       c)  $\frac{14^6}{7^6}$       d)  $\frac{99^3}{33^3}$       e)  $2^5 \frac{25^4}{5^4}$

a)  $2^4 \cdot 5^4 = (2 \cdot 5)^4 = 10^4 = 10\,000$

b)  $4^2 \cdot 5^2 = (4 \cdot 5)^2 = 20^2 = 400$

c)  $\frac{14^6}{7^6} = \left(\frac{14}{7}\right)^6 = 2^6 = 64$

d)  $\frac{99^3}{33^3} = \left(\frac{99}{33}\right)^3 = \left(\frac{3 \cdot 33}{33}\right)^3 = 3^3 = 27$

e)  $2^5 \frac{25^4}{5^4} = 2^5 \left(\frac{25}{5}\right)^4 = 2^5 \cdot 5^4 = 2 \cdot (2^4 \cdot 5^4) = 2 \cdot (2 \cdot 5)^4 = 2 \cdot 10^4 = 20\,000$

**Př. 6:** Zjednoduš na součin mocnin prvočísel.

a)  $6^2 \cdot 3$

b)  $2^3 \cdot 4^2 \cdot 8$

c)  $\frac{25 \cdot 15^2}{5^4}$

d)  $\frac{4^2 \cdot 8 \cdot 9}{12^2}$

a)  $6^2 \cdot 3 = (2 \cdot 3)^2 \cdot 3 = 2^2 \cdot 3^2 \cdot 3 = 2^2 \cdot 3^3$

b)  $2^3 \cdot 4^2 \cdot 8 = 2^3 \cdot (2 \cdot 2)^2 \cdot 2^3 = 2^3 \cdot 2^4 \cdot 2^3 = 2^{10}$

c)  $\frac{25 \cdot 15^2}{5^4} = \frac{5^2 \cdot (5 \cdot 3)^2}{5^4} = \frac{5^2 \cdot 5^2 \cdot 3^2}{5^4} = 3^2$

d)  $\frac{4^2 \cdot 8 \cdot 9}{12^2} = \frac{(2 \cdot 2)^2 \cdot 2^3 \cdot 3^2}{(2 \cdot 2 \cdot 3)^2} = \frac{2^2 \cdot 2^2 \cdot 2^3 \cdot 3^2}{2^2 \cdot 2^2 \cdot 3^2} = \frac{2^7}{2^4} = 2^3$

**Př. 7:** Zjisti, pro která  $n$  rovnost platí.

a)  $\frac{3^5}{3^3} = 3^n$

b)  $3^5 \cdot 3^2 = \frac{1}{3^n}$

c)  $\frac{a^n}{a^3} = a^7$

d)  $\frac{x^7}{x^n} = \frac{x^{n+1}}{x^2}$

a)  $\frac{3^5}{3^3} = 3^n$

$3^2 = 3^n$

$n = 2$

b)  $3^5 \cdot 3^2 = \frac{1}{3^n}$

$3^7 = 3^{-n}$

$7 = -n$

$n = -7$

c)  $\frac{a^n}{a^3} = a^7$

$a^{n-3} = a^7$

$n - 3 = 7 \quad / +3$

$n = 10$

d)  $\frac{x^7}{x^n} = \frac{x^{n+1}}{x^2}$

$x^{7-n} = x^{n-1}$

$7 - n = n - 1 \quad / +n + 1$

$8 = 2n \quad / : 2$

$n = 4$

**Shrnutí:**  $2^{-3} = \frac{1}{2^3}$