

3.5.10 Sčítání a odčítání lomených výrazů III

Předpoklady: 030509

Př. 1: Vypočti. Uveď podmínky.

a) $\frac{3a}{a+1} - \frac{a-2}{a+1}$ b) $\frac{a+2}{a-1} + \frac{2a+1}{1-a}$ c) $2 + \frac{1}{x} - \frac{1}{y}$

a) $\frac{3a}{a+1} - \frac{a-2}{a+1} = \frac{3a - (a-2)}{a+1} = \frac{3a - a + 2}{a+1} = \frac{2a+2}{a+1} = \frac{2(a+1)}{a+1} = 2$ $a \neq -1$

b) $\frac{a+2}{a-1} + \frac{2a+1}{1-a} = \frac{a+2}{a-1} + \frac{2a+1}{-(-1+a)} = \frac{a+2}{a-1} - \frac{2a+1}{a-1} = \frac{a+2-2a-1}{a-1} = \frac{1-a}{a-1} = \frac{-(a-1)}{a-1} = -1$
 $a \neq 1$

c) $2 + \frac{1}{x} - \frac{1}{y} = \frac{2 \cdot xy}{xy} + \frac{1 \cdot y}{xy} - \frac{1 \cdot x}{xy} = \frac{2xy + y - x}{xy}$ $x \neq 0; y \neq 0$

Př. 2: Vypočti. Uveď podmínky.

a) $\frac{x+1}{x} + 2$ b) $1 + a - \frac{1}{a}$ c) $y + \frac{2y}{y+1}$ d) $x - 1 + \frac{3x+1}{x+1}$

a) $\frac{x+1}{x} + 2 = \frac{x+1}{x} + \frac{2 \cdot x}{x} = \frac{x+1+2x}{x} = \frac{3x+1}{x}$ $x \neq 0$

b) $1 + a - \frac{1}{a} = \frac{(1+a) \cdot a}{a} - \frac{1}{a} = \frac{a^2 + a - 1}{a}$ $a \neq 0$

c) $y + \frac{2y}{y+1} = \frac{y(y+1)}{y+1} + \frac{2y}{y+1} = \frac{y^2 + y + 2y}{y+1} = \frac{y^2 + 3y}{y+1}$ $y \neq -1$

d) $x - 1 + \frac{3x+1}{x+1} = \frac{(x-1)(x+1)}{x+1} + \frac{3x+1}{x+1} = \frac{x^2 - 1}{x+1} + \frac{3x+1}{x+1} = \frac{x^2 - 1 + 3x + 1}{x+1} = \frac{x^2 + 3x}{x+1}$
 $x \neq -1$

Př. 3: Koukej to spočítat nebo uvidíš! A nezapomeň a podmínky!

a) $\frac{2}{x+1} + \frac{3}{x-2}$ b) $\frac{1+a}{2a-3} - \frac{1}{a}$ c) $\frac{y+1}{y-2} - \frac{y-3}{y+3}$ d) $\frac{x-1}{2x+1} + \frac{2x-1}{3x-1}$

a) $\frac{2}{x+1} + \frac{3}{x-2} = \frac{2(x-2) + 3(x+1)}{(x+1)(x-2)} = \frac{2x - 4 + 3x + 3}{(x+1)(x-2)} = \frac{5x-1}{(x+1)(x-2)}$ $x \neq -1; 2$

$$b) \frac{1+a}{2a-3} - \frac{1}{a} = \frac{(1+a) \cdot a - 1(2a-3)}{(2a-3)a} = \frac{a^2 + a - 2a + 3}{(2a-3)a} = \frac{a^2 - a + 3}{(2a-3)a} \quad a \neq 0; \frac{3}{2}$$

$$c) \frac{y+1}{y-2} - \frac{y-3}{y+3} = \frac{(y+1)(y+3) - (y-3)(y-2)}{(y-2)(y+3)} = \frac{y^2 + 3y + y + 3 - (y^2 - 2y - 3y + 6)}{(y-2)(y+3)} =$$

$$= \frac{y^2 + 4y + 3 - (y^2 - 5y + 6)}{(y-2)(y+3)} = \frac{9y - 3}{(y-2)(y+3)} = \frac{3(3y-1)}{(y-2)(y+3)}$$

$$y \neq -3; 2$$

$$d) \frac{x-1}{2x+1} + \frac{2x-1}{3x-1} = \frac{(x-1)(3x-1) + (2x-1)(2x+1)}{(2x+1)(3x-1)} = \frac{3x^2 - x - 3x + 1 + 4x^2 - 2x + 2x - 1}{(2x+1)(3x-1)} =$$

$$= \frac{7x^2 - 4x}{(2x+1)(3x-1)} = \frac{x(7x-4)}{(2x+1)(3x-1)}$$

$$x \neq -\frac{1}{2}; \frac{1}{3}$$

Př. 4: Zapiš podmínky a vypočti.

$$a) \frac{x+1}{x^2+3x} + \frac{x-1}{x+3} \quad b) \frac{a+3}{a^2+2a} + \frac{2}{a+2} \quad c) \frac{3}{y^2+y} - \frac{2}{y^2-y}$$

$$a) \frac{x+1}{x^2+3x} + \frac{x-1}{x+3} = \frac{x+1}{x(x+3)} + \frac{x(x-1)}{x(x+3)} = \frac{x+1+x^2-x}{x(x+3)} = \frac{x^2+1}{x(x+3)} \quad x \neq 0; -3$$

$$b) \frac{a+3}{a^2+2a} + \frac{2}{a+2} = \frac{a+3}{a(a+2)} + \frac{2}{a+2} = \frac{a+3+2a}{a(a+2)} = \frac{a+3+2a}{a(a+2)} = \frac{3a+3}{a(a+2)}$$

$$a \neq 0; -2$$

$$c) \frac{3}{y^2+y} - \frac{2}{y^2-y} = \frac{3}{(y+1)y} - \frac{2}{(y-1)y} = \frac{3(y-1) - 2(y+1)}{(y+1)y(y-1)} = \frac{3y-3-2y-2}{(y+1)y(y-1)} = \frac{y-5}{(y+1)y(y-1)}$$

$$y \neq -1; 0; 1$$

Př. 5: Zapiš podmínky a vypočti.

$$a) \frac{2x}{x^2-9} + \frac{x+1}{x^2+3x} \quad b) \frac{a-1}{a^2+2a} + \frac{a+3}{a^2+4a+4} \quad c) \frac{y-1}{y^2+y} - \frac{y+2}{y^2-y-2}$$

$$a) \frac{2x}{x^2-9} + \frac{x+1}{x^2+3x} = \frac{2x}{(x-3)(x+3)} + \frac{x+1}{x(x+3)} = \frac{2x \cdot x + (x+1)(x-3)}{x(x-3)(x+3)} =$$

$$= \frac{2x^2 + x^2 - 3x + x - 3}{x(x-3)(x+3)} = \frac{3x^2 - 2x - 3}{x(x-3)(x+3)}$$

$$x \neq -3; 0; 3$$

$$\frac{a-1}{a^2+2a} + \frac{a+3}{a^2+4a+4} = \frac{a-1}{(a+2)a} + \frac{a+3}{(a+2)^2} = \frac{(a-1)(a+2)+a(a+3)}{a(a+2)^2} =$$

b)

$$= \frac{a^2+2a-a-2+a^2+3a}{a(a+2)^2} = \frac{2a^2+4a-2}{a(a+2)^2}$$

$$a \neq -2; 0$$

$$\frac{y-1}{y^2+y} - \frac{y+2}{y^2-y-2} = \frac{y-1}{(y+1)y} - \frac{y+2}{(y-2)(y+1)} = \frac{(y-1)(y-2)-y(y+2)}{(y+1)y(y-2)} =$$

c)

$$= \frac{y^2-2y-y+2-(y^2+2y)}{(y+1)y(y-2)} = \frac{y^2-3y+2-y^2-2y}{(y+1)y(y-2)} = \frac{-5y+2}{(y+1)y(y-2)}$$

$$y \neq -1; 0; 2$$

Shrnutí: