

3.8.12 Sčítání lomených výrazů V

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Př. 1: Vypočti. Podmínky udělej taky.

$$\text{a) } a - \frac{2a+1}{a-2} \quad \text{b) } a + \frac{1}{a} - \frac{1}{b} \quad \text{c) } \frac{2}{x-1} - \frac{3}{x+2}$$

$$\text{a) } a - \frac{2a+1}{a-2} = \frac{a(a-2)}{a-2} - \frac{2a+1}{a-2} = \frac{a^2 - 2a - 2a - 1}{a-2} = \frac{a^2 - 4a - 1}{a-2} \quad a \neq 2$$

$$\text{b) } a + \frac{1}{a} - \frac{1}{b} = \frac{a \cdot ab}{ab} + \frac{1 \cdot b}{a \cdot b} - \frac{1 \cdot a}{b \cdot a} = \frac{a^2 b + b - a}{ab} \quad a \neq 0; b \neq 0$$

$$\text{c) } \frac{2}{x-1} - \frac{3}{x+2} = \frac{2(x+2)}{(x-1)(x+2)} - \frac{3(x-1)}{(x+2)(x-1)} = \frac{2x+4-3x+3}{(x-1)(x+2)} = \frac{7-x}{(x-1)(x+2)} \quad x \neq -2; 1$$

Př. 2: Zapiš podmínky a zjednoduš.

$$\text{a) } \frac{2x}{x(x+1)} + \frac{x+3}{x^2} \quad \text{b) } \frac{b+1}{a(a-b)} + \frac{a-1}{2b(a-b)} \quad \text{c) } \frac{x+3}{(x-2)(x-1)} - \frac{x+2}{(x-1)(x-3)}$$

$$\text{a) } \frac{2x}{x(x+1)} + \frac{x+3}{x^2} = \frac{2x \cdot x}{x(x+1) \cdot x} + \frac{(x+3)(x+1)}{x^2(x+1)} = \frac{2x^2 + x^2 + x + 3x + 3}{x^2(x+1)} = \frac{3x^2 + 4x + 3}{x^2(x+1)}$$

$$x \neq -1; 0$$

$$\text{b) } \frac{b+1}{a(a-b)} + \frac{a-1}{2b(a-b)} = \frac{(b+1) \cdot 2b}{a(a-b) \cdot 2b} + \frac{(a-1) \cdot a}{2b(a-b) \cdot a} = \frac{2b^2 + 2b + a^2 - a}{2ab(a-b)}$$

$$a \neq 0; b \neq 0; a \neq b$$

$$\text{c) } \frac{x+3}{(x-2)(x-1)} - \frac{x+2}{(x-1)(x-3)} = \frac{(x+3)(x-3)}{(x-2)(x-1)(x-3)} - \frac{(x+2)(x-2)}{(x-1)(x-2)(x-3)} =$$

$$= \frac{x^2 - 9 - (x^2 - 4)}{(x-2)(x-1)(x-3)} = \frac{-5}{(x-2)(x-1)(x-3)}$$

$$x \neq 1; 2; 3$$

Př. 3: Sčítej, sčítej sčítači (a podmínkuj).

$$\text{a) } \frac{a+1}{a^2+a} - \frac{2}{a+1} \quad \text{b) } \frac{3}{x-y} - \frac{3x+y}{2x^2-2xy} \quad \text{c) } \frac{a+1}{a^2-2a} + \frac{a-1}{a^2+a}$$

$$\text{a) } \frac{a+1}{a^2+a} - \frac{2}{a+1} = \frac{a+1}{(a+1)a} - \frac{2}{a+1} = \frac{a+1}{(a+1)a} - \frac{2 \cdot a}{(a+1) \cdot a} = \frac{a+1-2a}{(a+1)a} = \frac{1-a}{(a+1)a}$$

$$a \neq -1; 0$$

$$\text{b) } \frac{3}{x-y} - \frac{3x+y}{2x^2-2xy} = \frac{3}{x-y} - \frac{3x+y}{2x(x-y)} = \frac{3 \cdot 2x}{(x-y) \cdot 2x} - \frac{3x+y}{2x(x-y)} = \frac{6x-3x-y}{2x(x-y)} = \frac{3x-y}{2x(x-y)}$$

$x \neq 0; x \neq y$

$$\text{c) } \frac{a+1}{a^2-2a} + \frac{a-1}{a^2+a} = \frac{a+1}{(a-2)a} + \frac{a-1}{(a+1)a} = \frac{(a+1)^2}{(a-2)a(a+1)} + \frac{(a-1)(a-2)}{(a+1)a(a-2)} =$$

$$= \frac{a^2+2a+1+a^2-2a-a+2}{(a-2)a(a+1)} = \frac{2a^2-a+3}{(a-2)a(a+1)}$$

$a \neq -1; 0; 2$

Př. 4: Kdo nesečte (včetně podmínek), ať nejlí.

$$\text{a) } \frac{4}{x^2+2x} - \frac{2}{x^2-4} \quad \text{b) } \frac{3}{a^2+4a+4} - \frac{2}{a^2+3a+2} \quad \text{c) } \frac{4y}{y^2-y-6} + \frac{3y}{9-y^2}$$

$$\text{a) } \frac{4}{x^2+2x} - \frac{2}{x^2-4} = \frac{4}{(x+2)x} - \frac{2}{(x-2)(x+2)} = \frac{4(x-2)}{(x+2)x(x-2)} - \frac{2 \cdot x}{(x-2)(x+2) \cdot x} =$$

$$= \frac{4x-8-2x}{(x+2)x(x-2)} = \frac{2x-8}{(x+2)x(x-2)}$$

$a \neq -2; 0; 2$

$$\text{b) } \frac{3}{a^2+4a+4} - \frac{2}{a^2+3a+2} = \frac{3}{(a+2)^2} - \frac{2}{(a+2)(a+1)} = \frac{3(a+1)}{(a+1)(a+2)^2} - \frac{2(a+2)}{(a+2)^2(a+1)} =$$

$$= \frac{3a+3-2a-4}{(a+1)(a+2)^2} = \frac{a-1}{(a+1)(a+2)^2}$$

$a \neq -1; -2$

$$\text{c) } \frac{4y}{y^2-y-6} + \frac{3y}{9-y^2} = \frac{4y}{(y-3)(y+2)} + \frac{3y}{(3-y)(3+y)} = \frac{4y}{(y-3)(y+2)} + \frac{3y}{-(y-3)(3+y)} =$$

$$= \frac{4y(y+3)}{(y-3)(y+2)(y+3)} - \frac{3y(y+2)}{(y-3)(3+y)(y+2)} = \frac{4y^2+12y-(3y^2+6y)}{(y-3)(y+3)(y+2)} = \frac{y^2-6y}{(y-3)(y+3)(y+2)}$$

$y \neq -2; 2; 3$

Shrnutí: